Hybrid tractable CSPs which generalize tree structure

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Hybrid tractable CSPs which generalize tree structure

Constraint satisfaction

CSP = constraint satisfaction problem

Constraint satisfaction

assign values to variables to satisfy constraints

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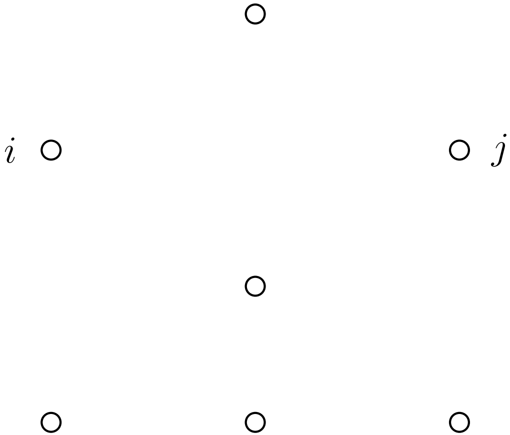
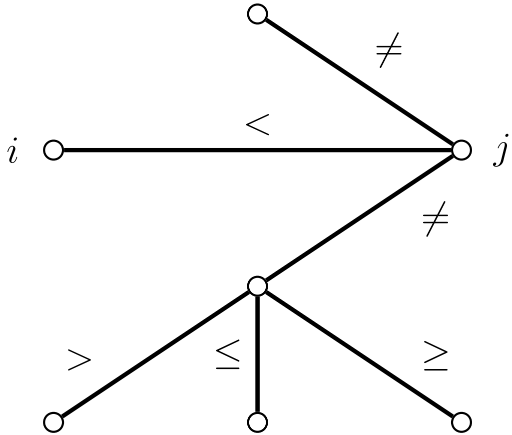


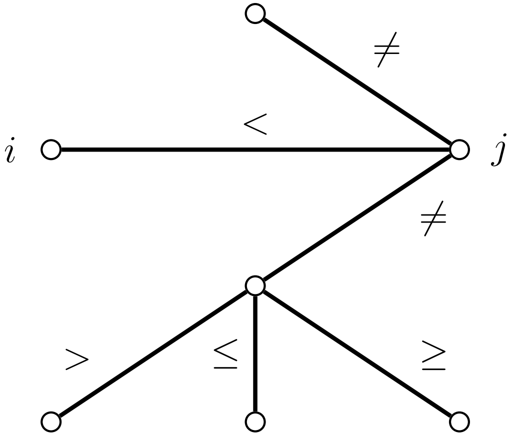
Constraint satisfaction

assign values to variables to satisfy constraints



Constraint satisfaction

binary constraint network



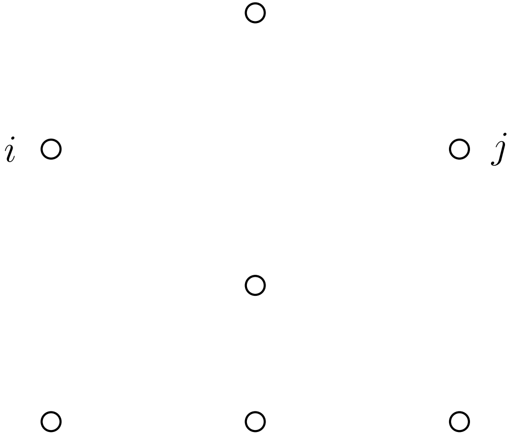
Constraint satisfaction

. . . is expressive:

SCHEDULING
k-SAT
INTEGER LINEAR PROGRAMMING
⋮

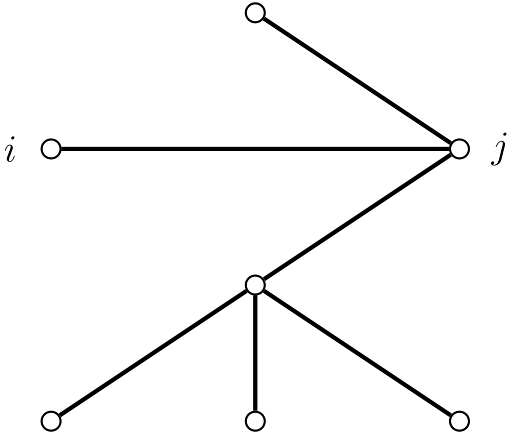
Hybrid tractable CSPs which generalize tree structure

Structure



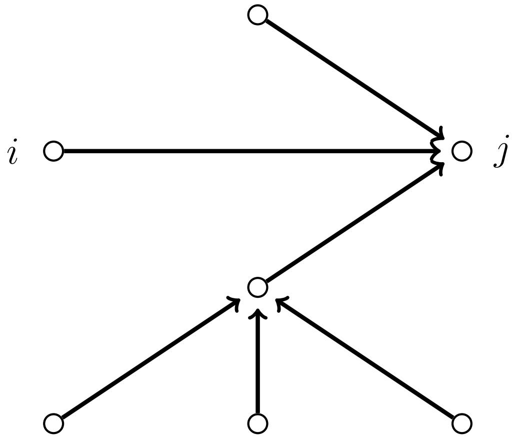
Structure

structure = constraint network's graph



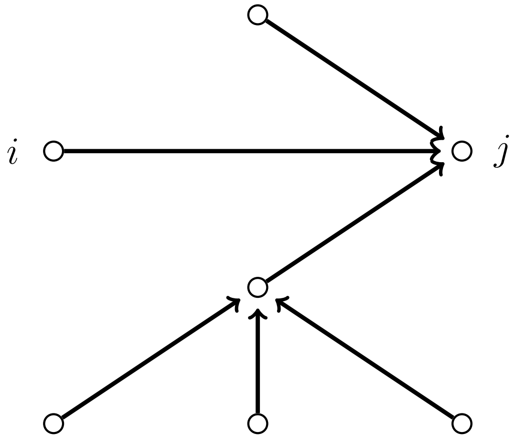
Structure

tree structure is tractable



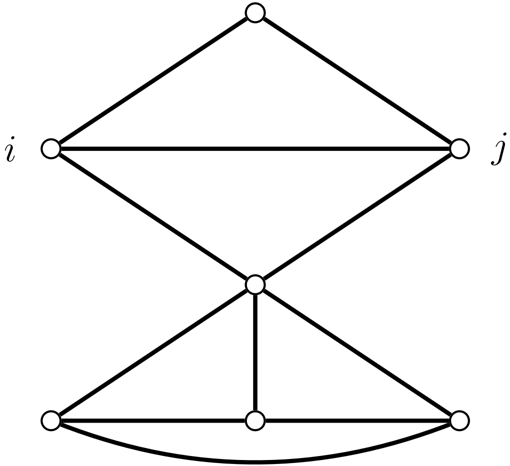
Structure

tree-like structure is tractable: Dechter/Pearl

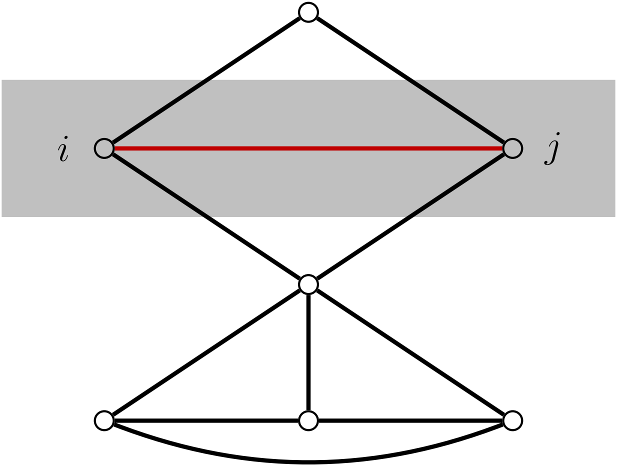


Structure

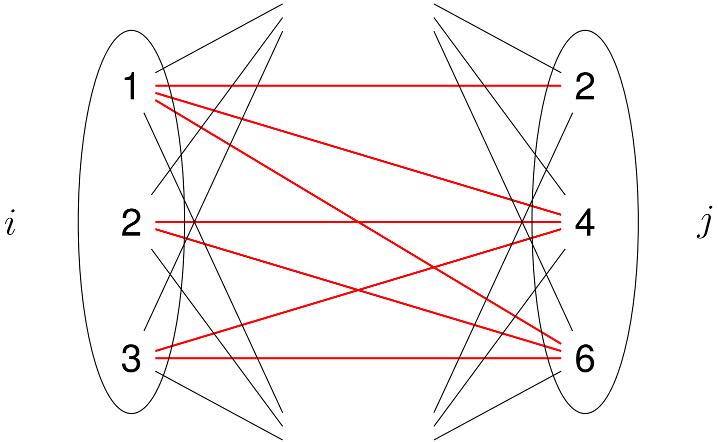
not tree-like structure: Grohe



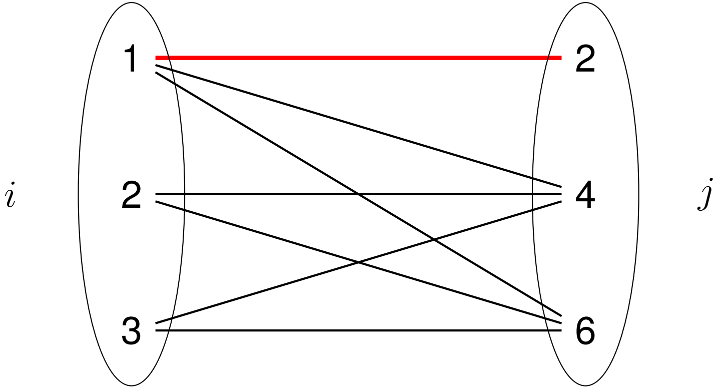
Structure



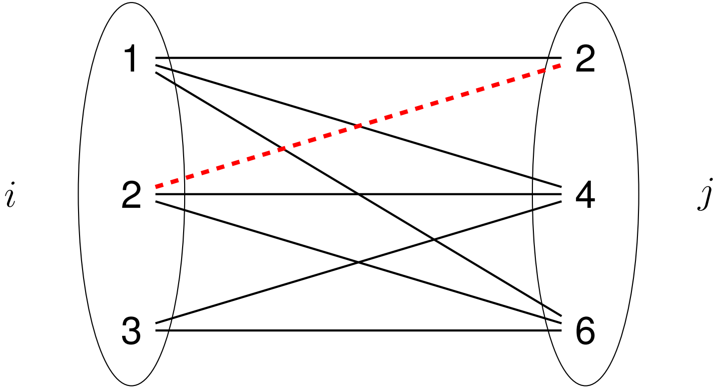
Language



Language

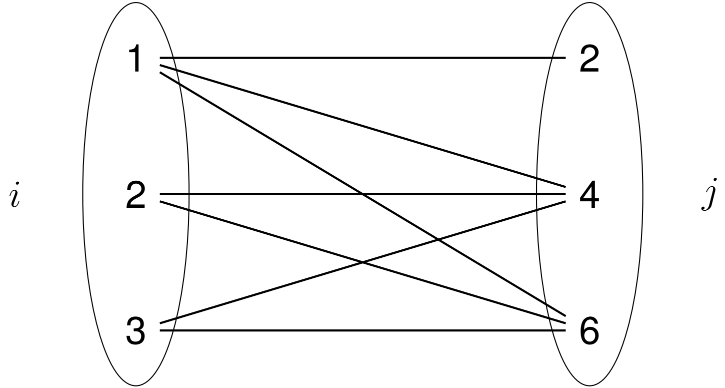


Language



Language

{ (1,2), (1,4), (1,6), (2,4), (2,6), (3,4), (3,6)}



Language

set of constraint types

$=$ set of constraint relations

$=$ constraint language

Language

tractable constraint languages:

$\{=\}$

max-closed

$\vdots$

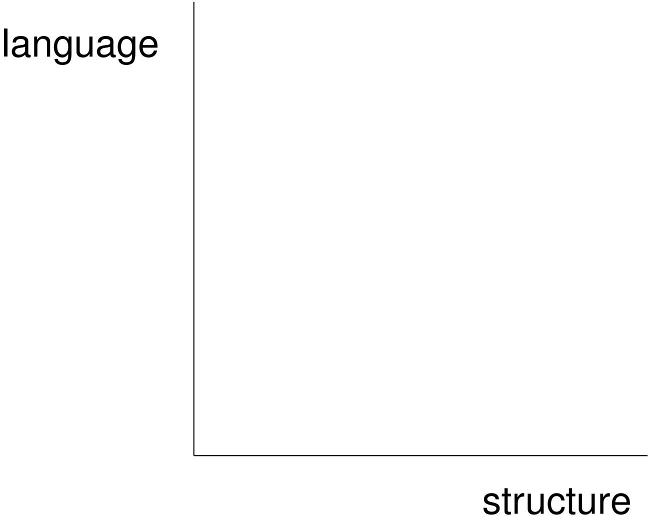
Tractability

- structural
- language
- other reasons?

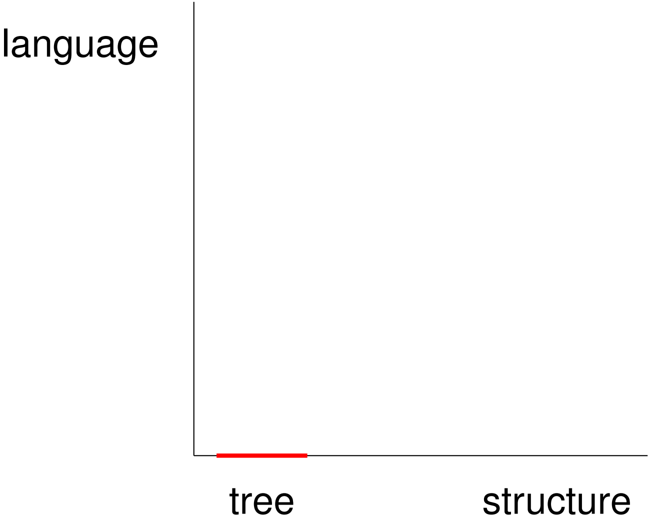
Hybrid tractable CSPs which generalize tree structure

Tractable CSPs

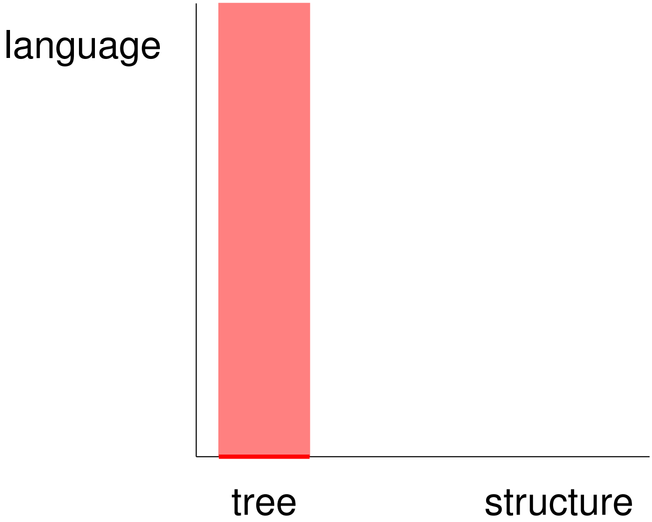
Tractable CSPs



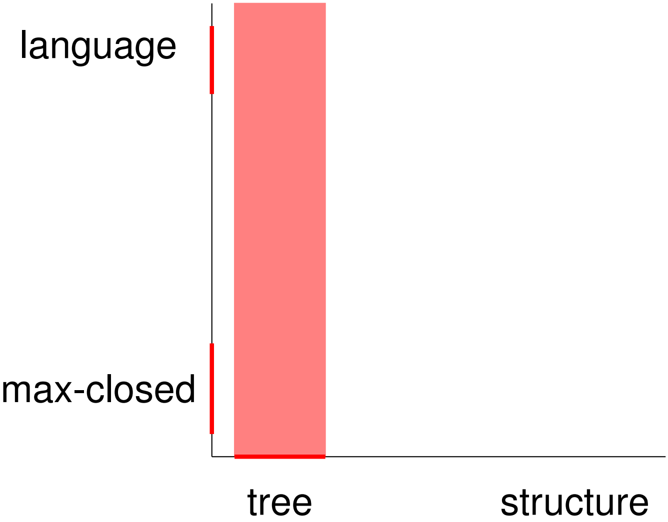
Tractable CSPs



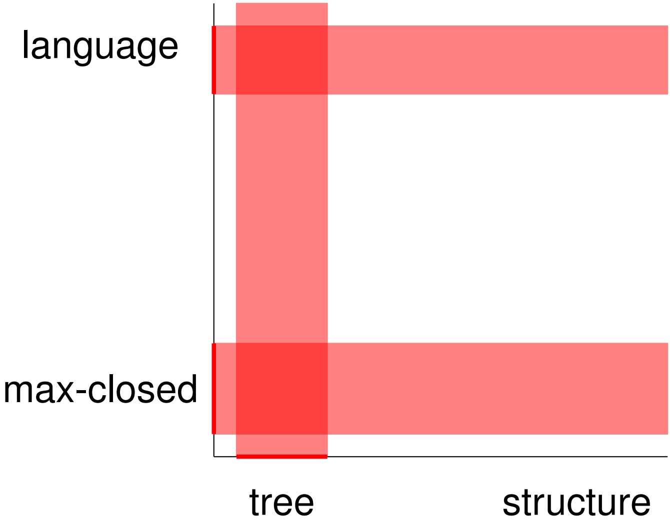
Tractable CSPs



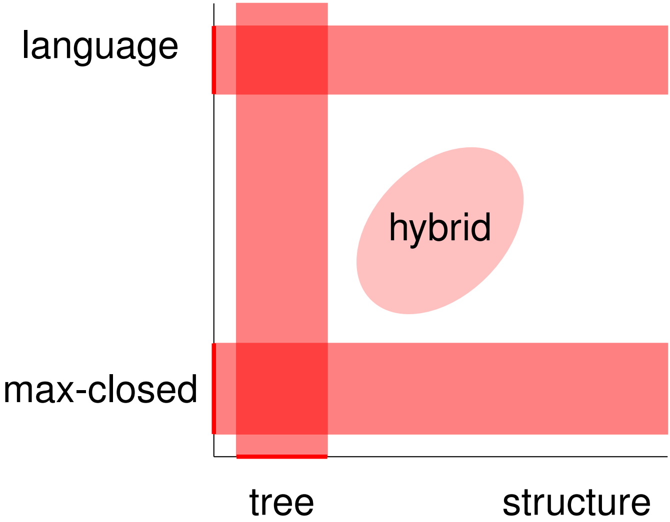
Tractable CSPs



Tractable CSPs

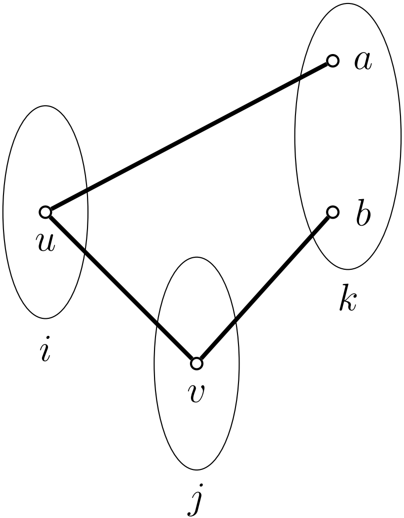


Tractable CSPs

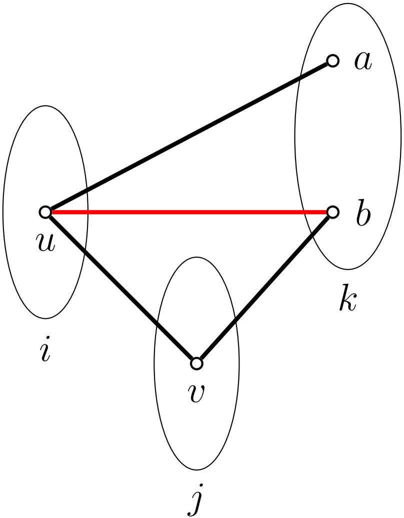


Hybrid tractable CSPs
which generalize tree structure

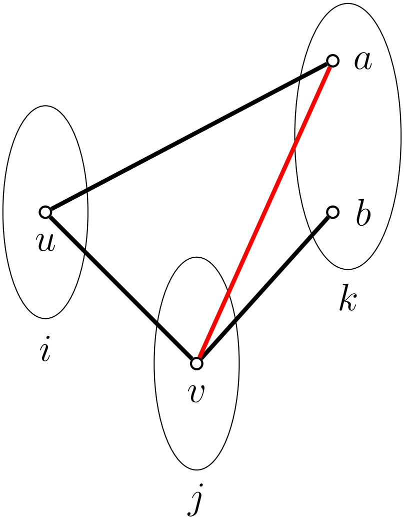
Broken-triangle property



Broken-triangle property



Broken-triangle property



Broken-triangle property

preserved by arc-consistency
(removing values from domains)

Algorithm 1

INPUT: variable ordering $<$ such that broken-triangle property holds w.r.t. $<$
OUTPUT: find solution
(or that there are no solutions)

BTP-SOLVE
1. establish arc consistency
2. follow $<$ and assign values

Algorithm 1

observation: backtrack-free
complexity: $O(d^2q)$
$\Rightarrow$ tractable to solve, given $<$

Algorithm 2

INPUT: given CSP instance
OUTPUT: find variable ordering $<$ such that broken-triangle property holds w.r.t. $<$ (or determine that no ordering exists)

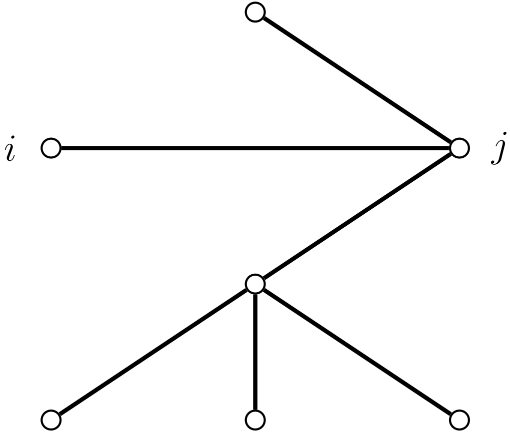
ORDERING-CSP
1. for all triples (i, j, k) of variables:
   if BTP obstruction on (i, j, k),
   then impose constraint $i, j \not< k$
2. solve this new CSP

Algorithm 2

observation: new CSP is max-closed
complexity: construct CSP in polynomial time
then solve CSP in polynomial time
$\Rightarrow$ tractable to find $<$

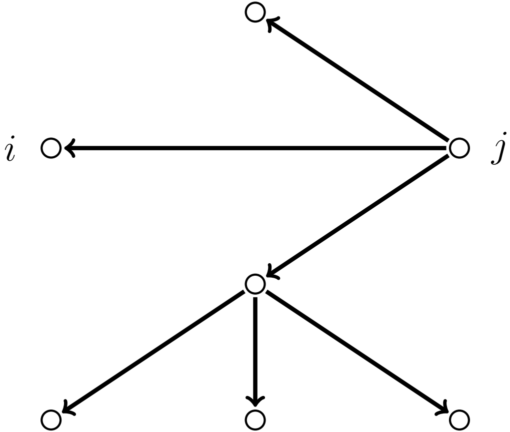
Hybrid tractable CSPs which generalize tree structure

Generalize tree structure



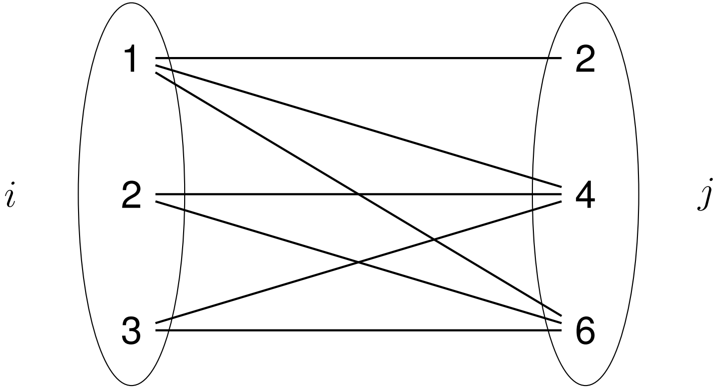
Generalize tree structure

Lemma 14: tree structure $\Rightarrow$ BTP



Renamable right monotone (RRM)

right monotone relation



Generalize RRM

Lemma 13: w.r.t. fixed $<$, RRM $\Rightarrow$ BTP

e.g. monotone constraints on grids
(so there are CSPs with unbounded treewidth
which also have broken-triangle property)

Other results

Theorem 15: TREE $\subsetneq$ BTP and RRM $\subsetneq$ BTP

Other results

Proposition 20: Let S be a conservative, inclusion-closed CSP with some variable ordering. The following are equivalent:
1. for every I in S, DAC(I) is universally backtrack-free
2. for every I in S, DAC(I) has the broken-triangle property.

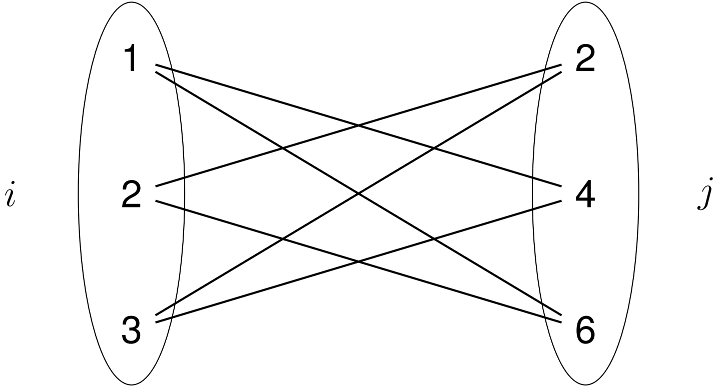
Hybrid tractable CSPs
which generalize tree structure

Further work

- better algorithm
- other hybrid properties ensuring tractability
- extension to non-binary CSPs

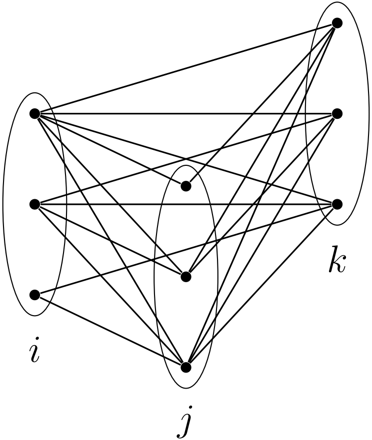
RRM vs. tree structure

TREE $\not\subseteq$ RRM

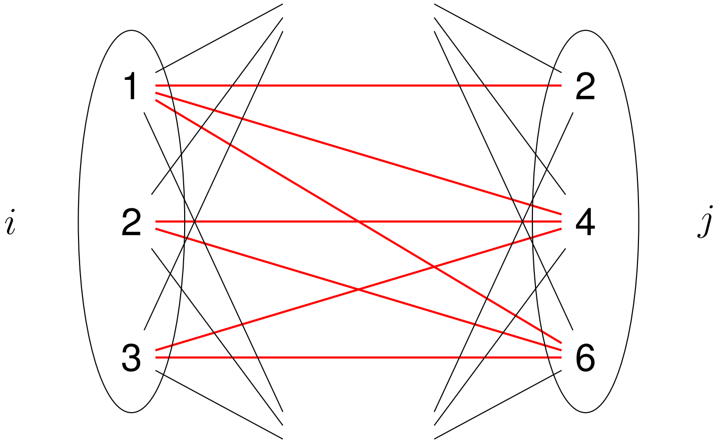


RRM vs. tree structure

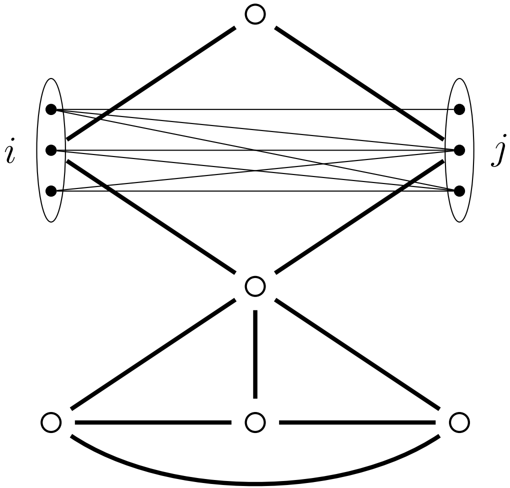
RRM $\subsetneq$ TREE: RRM w.r.t. $k < j < i$, not in TREE, not RRM w.r.t. $i < j < k$.



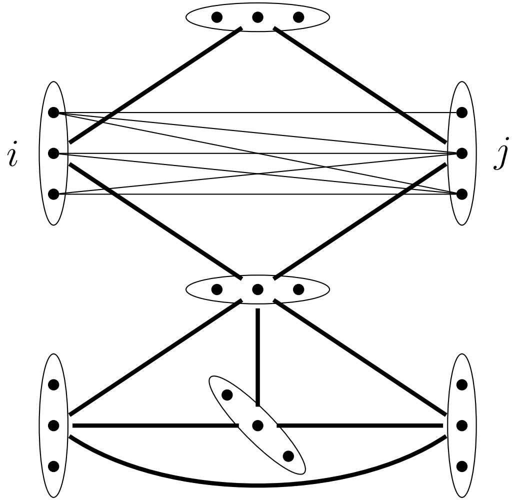
Beyond structure and language



Beyond structure and language



Beyond structure and language



Other results

min-of-max extendable: non-conservative generalization of BTP

> Proposition 26: directional path-consistent and row convex $\Rightarrow$ min-of-max extendable