Limits of diversification: Ibragimov and Walden

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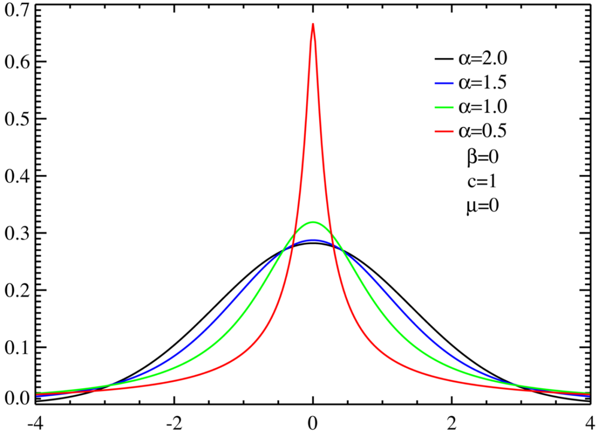
Historical context

- published 2007
- positioned as analysis of heavy-tailed phenomena
- recent papers with empirical studies
- (Fama 1965) evidence that inter-day changes in stock market prices follow stable distribution with $\alpha < 2$
- (Mandelbrot 1963) self-similar tails, e.g. cotton prices
- (Gnedenko and Kolmogorov 1954) limiting distributions of sums of independent identically distributed random variables are the stable ones, if they exist
- (Lévy 1920-1959) sums of random variables

Stable distributions



http://www.wikipedia.org/

Background

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Value at Risk of random variable Z at level q:

$$\mathrm{VaR}_q(Z) = \inf\{z \mid Pr[Z > z] \leq q\}$$

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- $n \geq 1$
- $\mathbf{w} = (w_1, w_2, \ldots, w_n)$, $\sum_{i=1}^{n} w_i = 1$
- $w_{[1]} \geq w_{[2]} \geq \ldots \geq w_{[n]} \geq 0$
- $X_{\mathbf{w}} = w_1 X_1 + w_2 X_2 + \ldots + w_n X_n$
- $w^{(1)} = \begin{cases} w_{[1]} & \text{if } w_{[1]} \geq 1/2 \\ 1/2 & \text{otherwise} \end{cases}$
- $w^{(2)} = \begin{cases} 1 - w_{[1]} & \text{if } w_{[1]} \geq 1/2 \\ 1/2 & \text{otherwise} \end{cases}$

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Degree of diversification: n

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class $CS(r)$ of distributions: If $Y_1, Y_2, \ldots, Y_k$ are independent with stable distributions with parameters $r < \alpha_i < 1$, $\beta = \mu = 0$, $\sigma_i > 0$, then $\sum_{i=1}^{k} Y_i \stackrel{\mathrm{d}}{=} D \in CS(r)$.

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Proposition 1. If X_i are iid with $X_i \stackrel{\mathrm{d}}{=} D_i \in CS(r)$ for $0 < r < 1$, $z > 0$, $w_{[1]} < 1$, then

$$Pr[X_{\mathbf{w}} > z] \geq Pr[w^{(1)} X_1 + w^{(2)} X_2 > z] > Pr[X_1 > z].$$

Main contribution

Theorem 1. If $n \geq 2$, $w_{[1]} < 1$, $z > 0$, $a > \tau(n, r)$ then $Pr[Y_{\mathbf{w}}(a) > z] > Pr[Y_1(a) > z]$.

- bounded risks: VaR first decreases with n, then increases
- need high degree of diversification for risk to be decreased
- note Proposition 1 is for unbounded case
- $Y(a)$ is X with cutoff: a outside interval $-a \leq z \leq a$
- $Y_{\mathbf{w}}(a) = \sum_{i=1}^{n} w_i Y_i(a)$
- Theorem 4 relaxes independence assumption

Taleb's stance

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- *Finiteness of Variance is Irrelevant in the Practice of Quantitative Finance* (draft, June 2008)
- need to incorporate serial dependence of absolute returns
- conclusion: assume power law (not necessarily stable)
- evidence that $\alpha \approx 3$
- infinite moments: cannot expand, cannot use Itō's Lemma
- suggestion: patch piecewise, discard elegance
- any merit?