

Hybrid tractable CSPs which generalize tree structure

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Hybrid tractable **CSPs** which generalize tree structure

Constraint satisfaction



CSP = constraint satisfaction problem

Constraint satisfaction

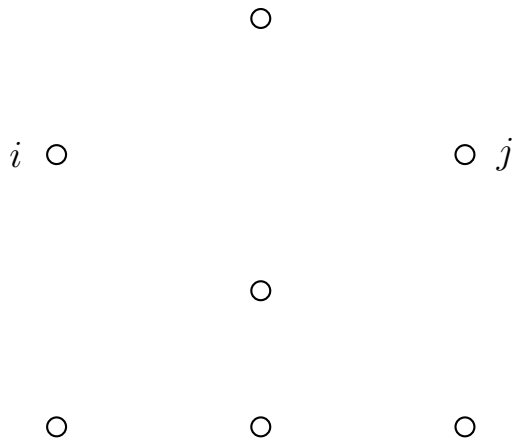


assign values to variables to satisfy constraints

Constraint satisfaction



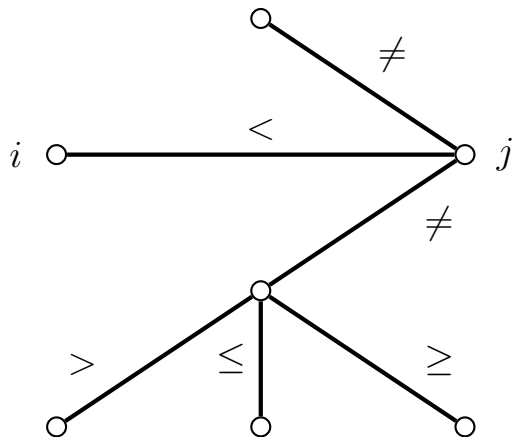
assign values to **variables** to satisfy constraints



Constraint satisfaction



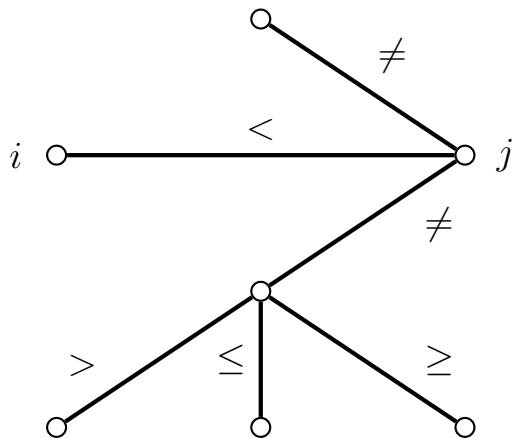
assign values to variables to **satisfy constraints**



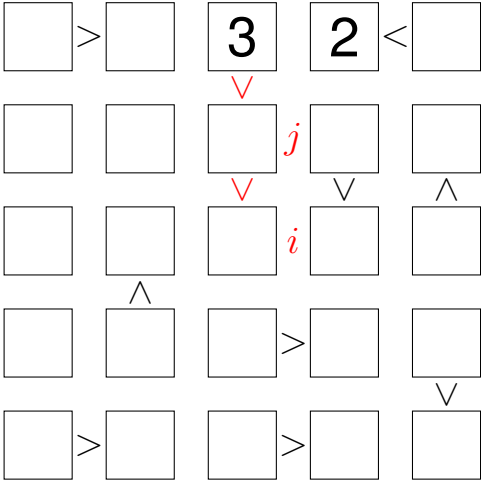
Constraint satisfaction



binary constraint network



Futoshiki



Constraint satisfaction



... is expressive:

SCHEDULING

SATISFIABILITY

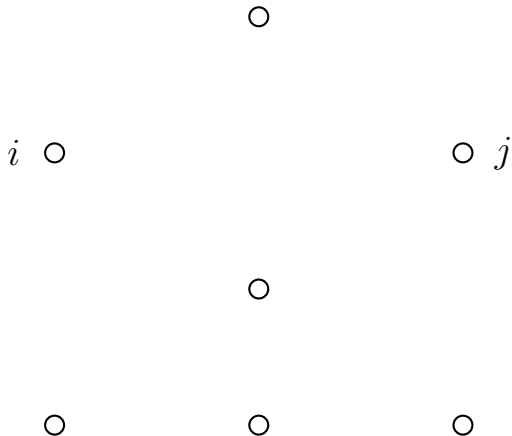
CONJUNCTIVE QUERY EVALUATION

INTEGER LINEAR PROGRAMMING

⋮

Hybrid **tractable** CSPs which generalize tree structure

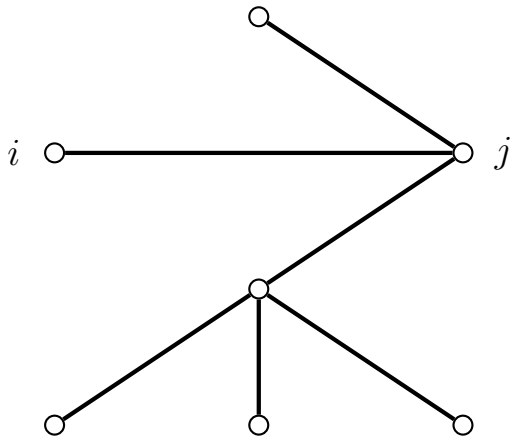
Structure



Structure

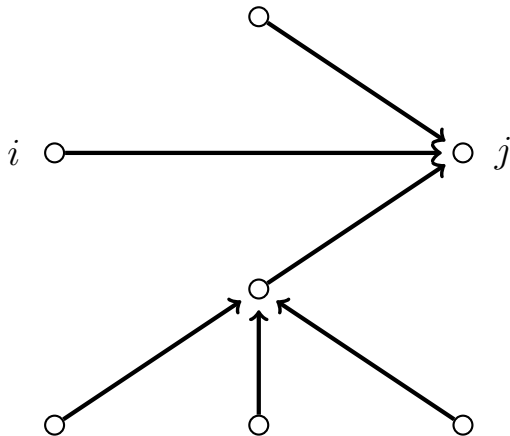


structure = constraint network's graph



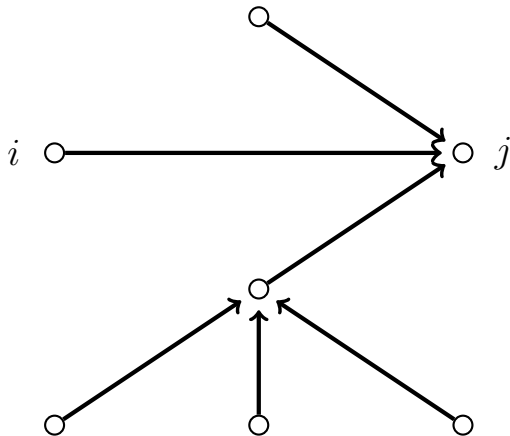
Structure

tree structure is tractable



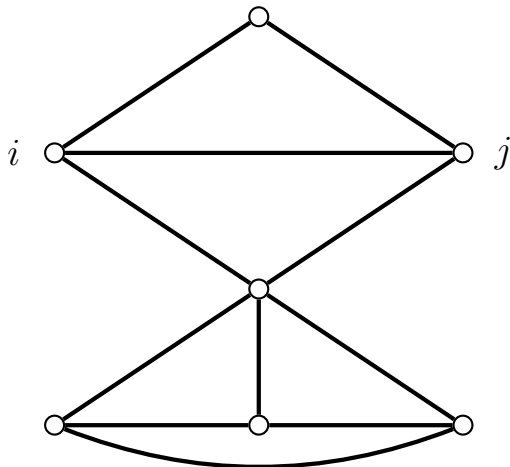
Structure

tree-like structure is tractable: Dechter/Pearl

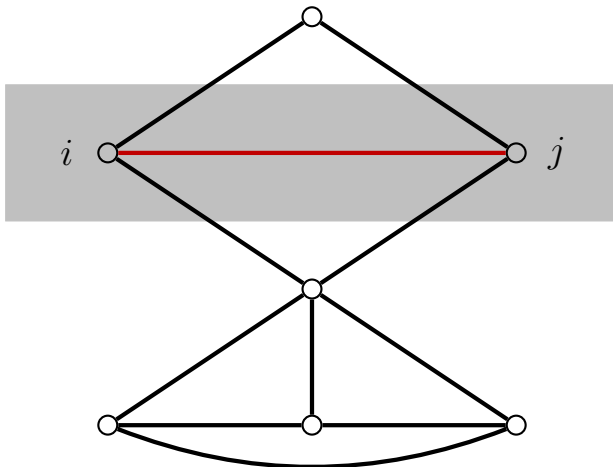


Structure

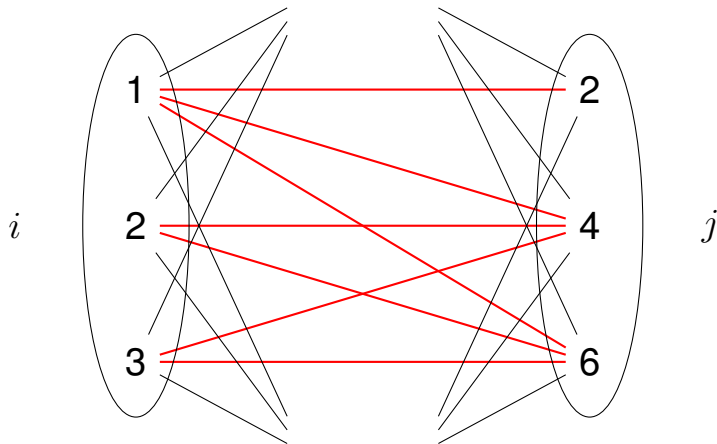
not tree-like structure: Grohe



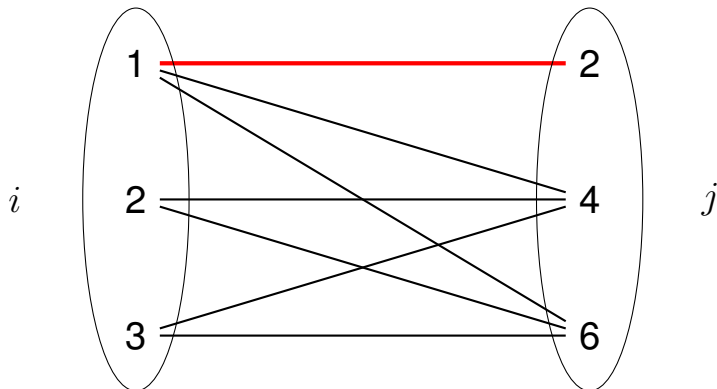
Structure



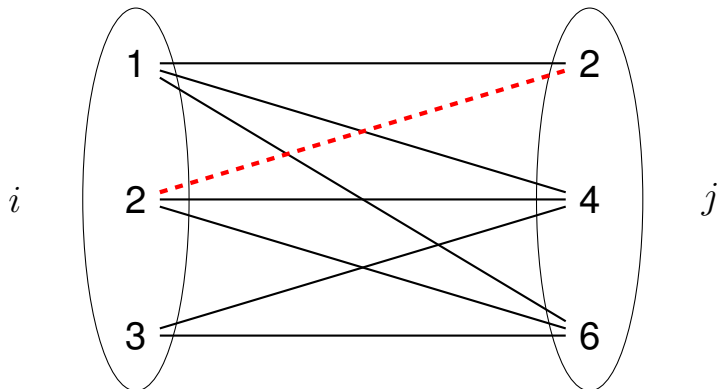
Language



Language

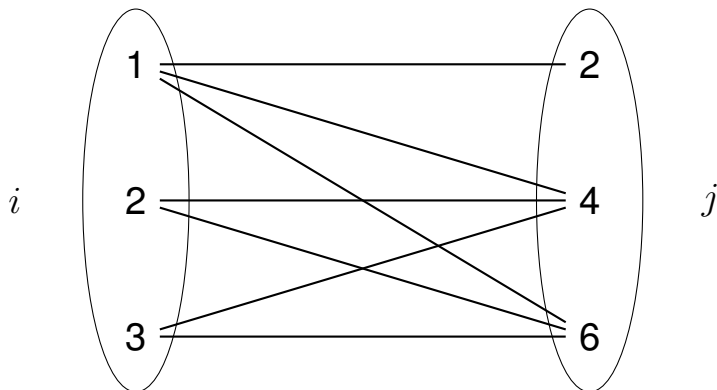


Language



Language

$\{ (1,2), (1,4), (1,6), (2,4), (2,6), (3,4), (3,6) \}$



Language



set of constraint types
= set of constraint relations
= constraint language

Language



tractable constraint languages:

$\{=\}$

max-closed

⋮

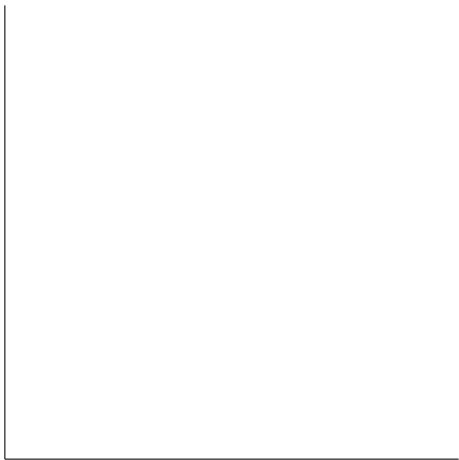
Tractability



- ▶ structural
- ▶ language
- ▶ other reasons?

Hybrid tractable CSPs which generalize tree structure

Tractable CSPs



CSP

Tractability

Hybrid

BTP

Generalize

Conclusion

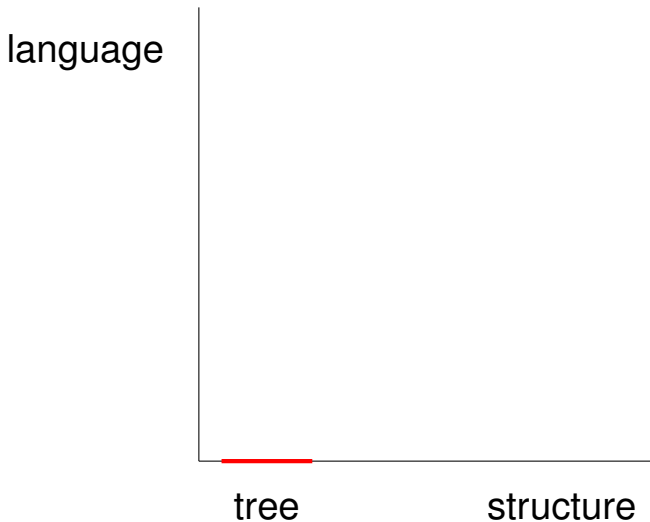
Tractable CSPs



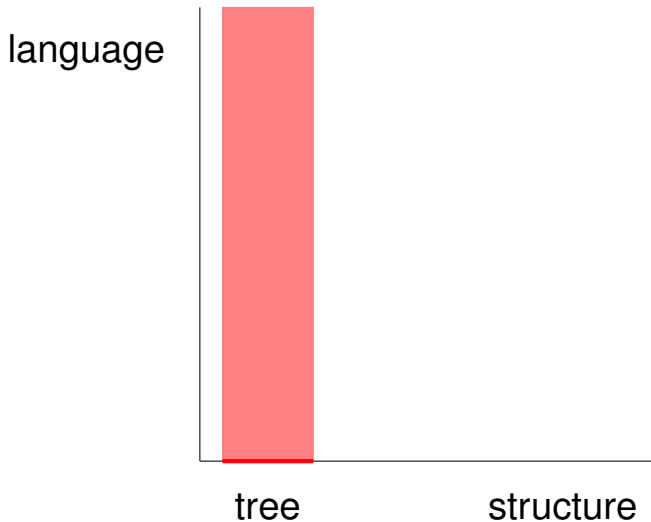
language

structure

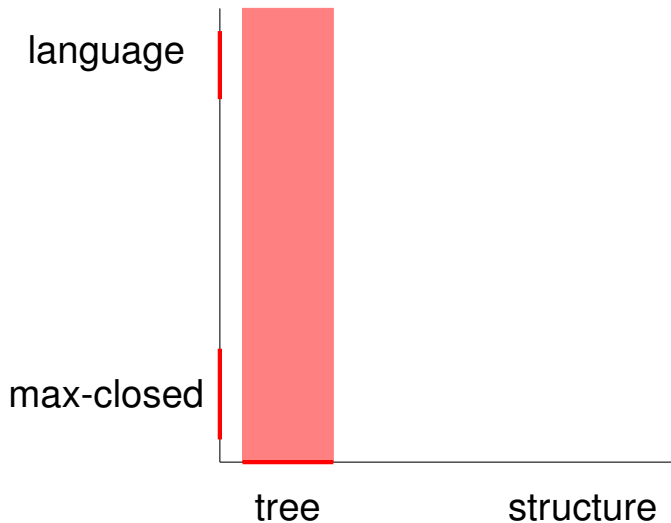
Tractable CSPs



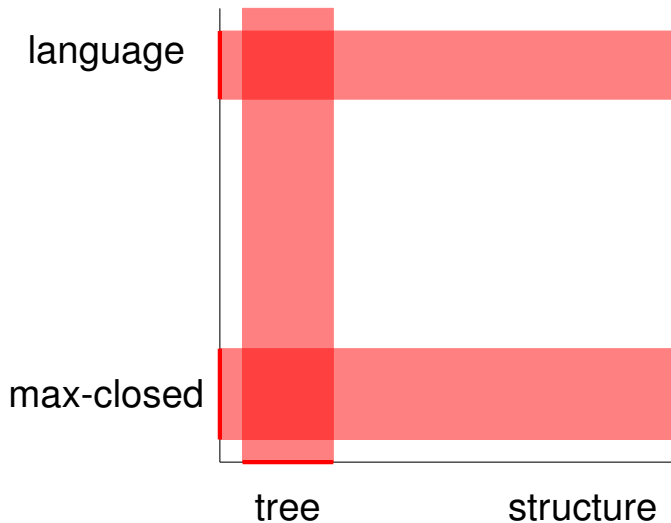
Tractable CSPs



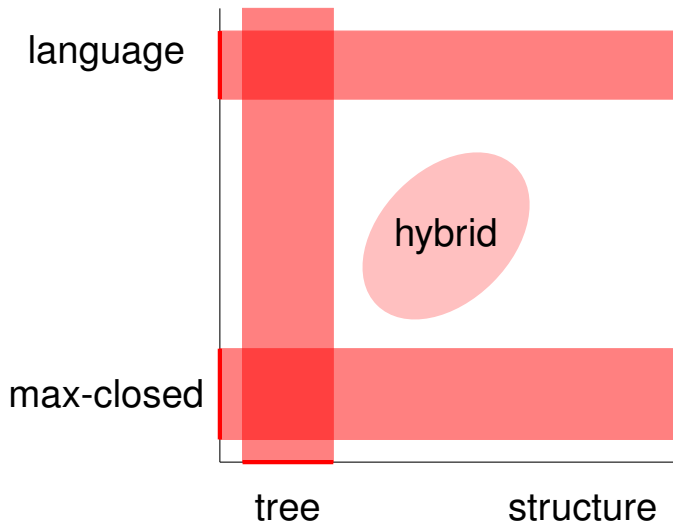
Tractable CSPs



Tractable CSPs

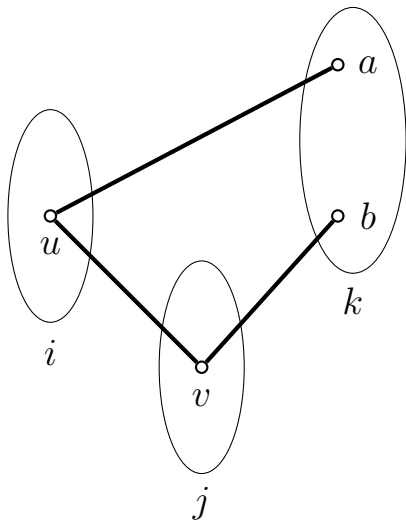


Tractable CSPs

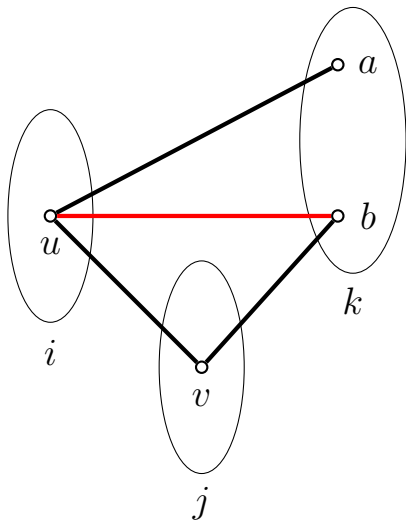


Hybrid tractable CSPs which generalize tree structure

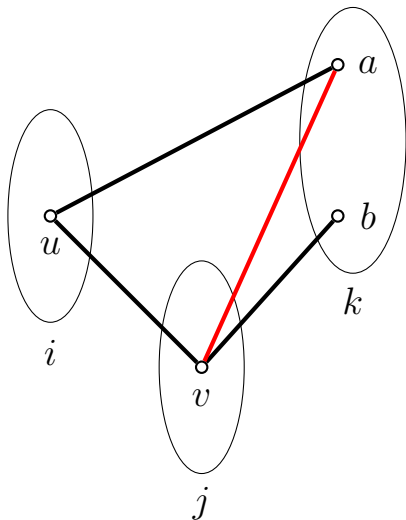
Broken-triangle property



Broken-triangle property



Broken-triangle property



Broken-triangle property



preserved by arc-consistency
(removing values from domains)

Algorithm 1



INPUT: variable ordering \prec such that
broken-triangle property holds w.r.t. \prec
OUTPUT: find solution
(or that there are no solutions)

BTP-SOLVE

1. establish arc consistency
2. follow \prec and assign values

Algorithm 1



observation: backtrack-free
complexity: $O(d^2q)$
 \Rightarrow tractable to solve, given $<$

Algorithm 2



INPUT: given CSP instance

OUTPUT: find variable ordering $<$ such that broken-triangle property holds w.r.t. $<$ (or determine that no ordering exists)

ORDERING-CSP

1. for all triples (i, j, k) of variables:
if BTP obstruction on (i, j, k) ,
then impose constraint $i, j \not\prec k$
2. solve this new CSP

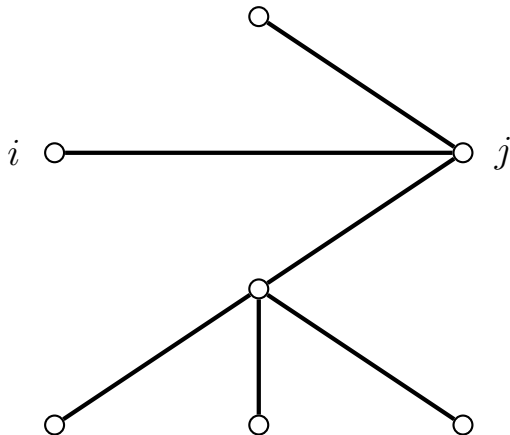
Algorithm 2



observation: new CSP is max-closed
complexity: construct CSP in polynomial time
then solve CSP in polynomial time
 \Rightarrow tractable to find \prec

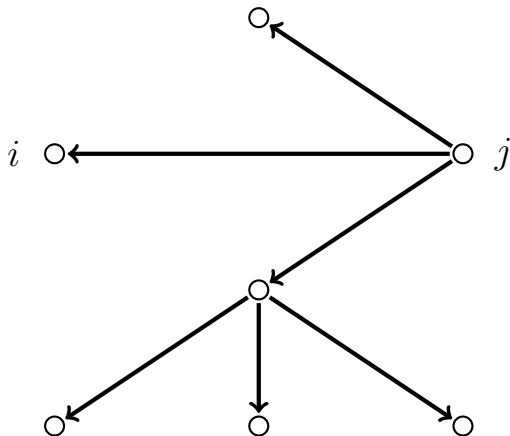
Hybrid tractable CSPs which generalize tree structure

Generalize tree structure



Generalize tree structure

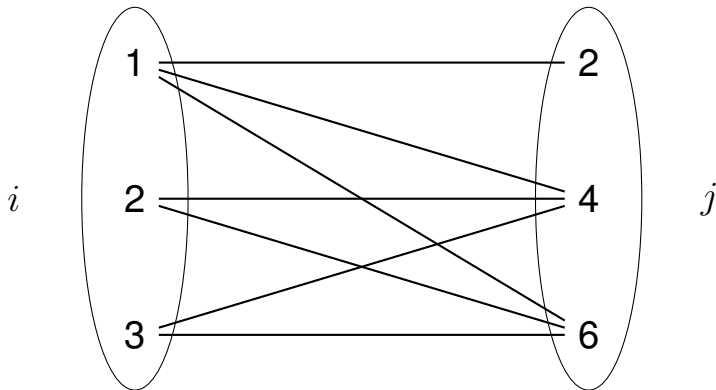
Lemma 14: tree structure \Rightarrow BTP



Renamable right monotone (RRM)



right monotone relation



Generalize RRM



Lemma 13: w.r.t. fixed $<$, $\text{RRM} \Rightarrow \text{BTP}$

e.g. monotone constraints on grids
(so there are CSPs with unbounded treewidth
which also have broken-triangle property)

Other results



Theorem 15: $\text{TREE} \not\subseteq \text{BTP}$ and $\text{RRM} \not\subseteq \text{BTP}$

Proposition 20: Let S be a conservative, inclusion-closed CSP with some variable ordering. The following are equivalent:

1. for every I in S , $\text{DAC}(I)$ is universally backtrack-free
2. for every I in S , $\text{DAC}(I)$ has the broken-triangle property.

Hybrid tractable CSPs which generalize tree structure

Further work

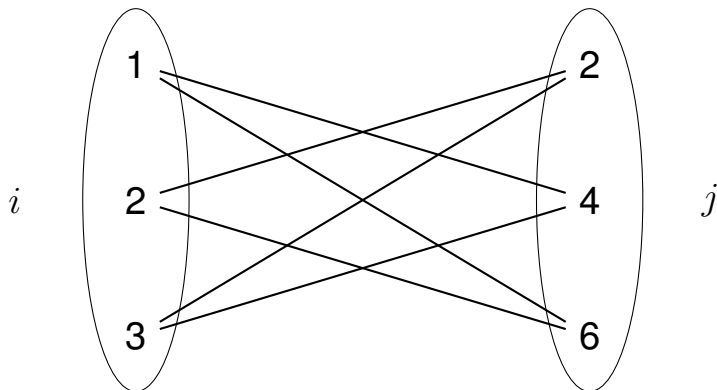


- ▶ better algorithm
 - ▶ other hybrid properties ensuring tractability
 - ▶ extension to non-binary CSPs
-

RRM vs. tree structure

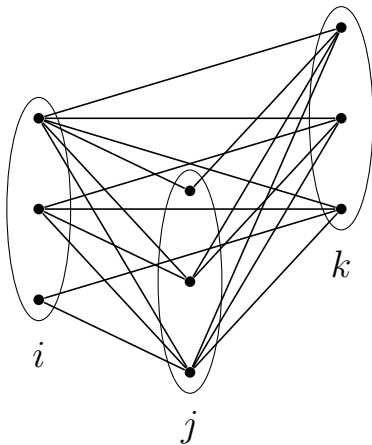


TREE $\not\subseteq$ RRM

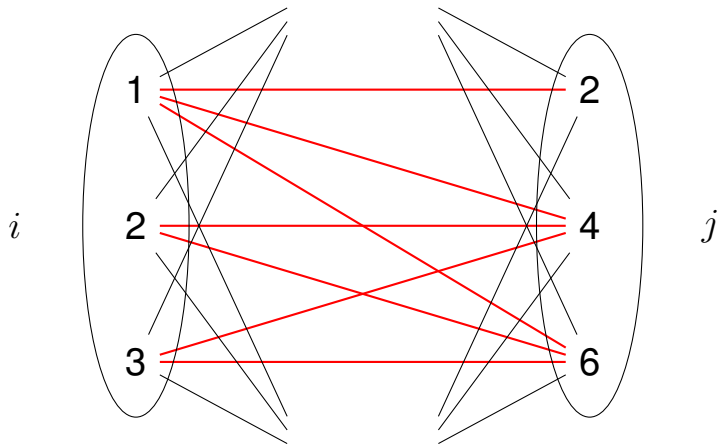


RRM vs. tree structure

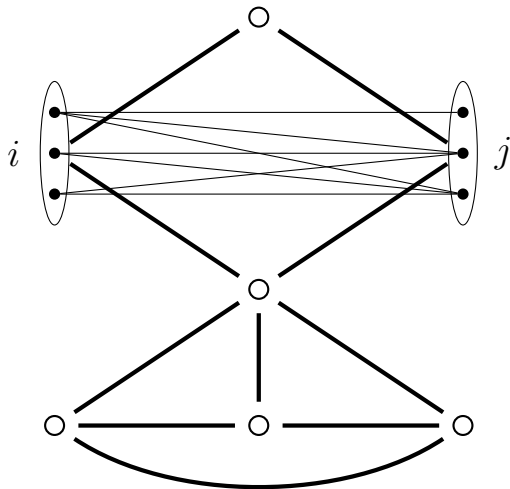
RRM \subsetneq TREE: RRM w.r.t. $k < j < i$, not in TREE, not RRM w.r.t. $i < j < k$.



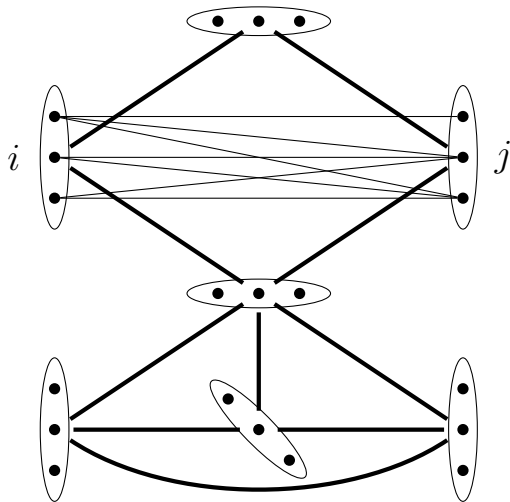
Beyond structure and language



Beyond structure and language



Beyond structure and language



min-of-max extendable: non-conservative
generalization of BTP

Proposition 26: directional path-consistent
and row convex \Rightarrow min-of-max extendable